

Organized by the Institute of Mathematics, University of Debrecen



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General Information

The conference takes place in Hotel Aurum in Hajdúszoboszló, Hungary, from Sunday, February 2 (arrival day) to Friday, February 7, 2025 (departure day). The participation fee covers full board accommodation, use of the spa and sauna in Hotel Aurum, registration materials and the fees of the banquet. The participants are put up in Hotel Aurum. Internet is available there with wireless connection.

All conference *talks* are given in *Park Hotel Ambrózia*, where the lecture room is equipped with a computer, a data projector, and a blackboard. The Coffee Breaks are also spent in that hotel. The allocated time for regular talks is 20 minutes at most and for plenary presentations it is maximum 40 minutes. The talks are followed by a discussion of at most 5 minutes. There are no breaks between the presentations within a session, therefore the individual talks are only approximately scheduled. Speakers cannot use the time gained from the shorter presentations, which then can be devoted to problems and remarks at the end of the session. If you have special wishes concerning the schedule, you are welcome to consult Gergő Nagy, the scientific secretary of the conference.

The program and the abstracts can be found in this booklet. Further technical details are announced during the conference. Your questions may assist the Organizing Committee in improving organization, so do not hesitate to contact us. We hope that our conference will be interesting and successful and you will enjoy your stay in Hajdúszoboszló.

Program

	Sunday		
18:00-20:00	Dinner		
	Monday		Tuesday
07:30-09:00	Breakfast	07:30-09:00	Breakfast
09:00-09:10	Opening		
09:10-10:20	1 st Morning Session	09:00-10:10	1 st Morning Session
10:20–10:50	Coffee Break	10:10-10:40	Coffee Break
10:50-12:00	2 nd Morning Session	10:40-11:55	2 nd Morning Session
12:00-13:00	Lunch	12:00-13:00	Lunch
15:00-16:15	1 st Afternoon Session	15:00-16:10	Afternoon Session
16:15–16:45	Coffee Break	16:10–16:40	Coffee Break
16:45–17:55	2 nd Afternoon Session	17:00-	Festive Dinner at
18:00-20:00	Dinner		University of Debrecen
	Wednesday		Thursday
07:30-09:00	Breakfast	07:30-09:00	Breakfast
09:00-10:10	1 st Morning Session	09:00-10:10	1 st Morning Session
10:10–10:40	Coffee Break	10:10-10:40	Coffee Break
10:40–11:55	2 nd Morning Session	10:40-11:55	2 nd Morning Session
12:00-13:00	Lunch	12:00-13:00	Lunch
15:00-16:15	1 st Afternoon Session	15:00-16:15	1 st Afternoon Session
16:15–16:45	Coffee Break	16:15–16:45	Coffee Break
16:45–17:55	2 nd Afternoon Session	16:45-18:00	2 nd Afternoon Session
18:00-20:00	Dinner	18:00-20:00	Dinner
	Friday		
07:30-09:00	Breakfast		

February 3, Monday

09:00–09:10 Opening

1st Morning Session – Chair: László Székelyhidi

09:10–09:50 Roman Ger: Erdős number
09:55–10:15 Attila Gilányi: On a functional equation arising from investigations of utility

Coffee Break 10:20–10:50

 2^{nd} Morning Session – Chair: Roman Ger

10:50-11:30	Zsolt Páles: Contributions of János Aczél to the theory of means
11:35-11:55	Gergely Kiss: Two theorems of Aczél and continuity

Lunch 12:00-13:00

 1^{st} Afternoon Session – Chair: Justyna Sikorska

15:00-15:20	Gian Luigi Forti: C^2 solutions of an alternative quadratic functional
	equation
15:25-15:45	Ioan Raşa: Functional equations related to Appell polynomials and
	Heun functions (II)
15:50-16:10	László Székelyhidi: Convolution-Type Functional Equations

Coffee Break 16:15-16:45

2nd Afternoon Session – Chair: Alina Ramona Baias

- 16:45–17:05 Karol Baron: Strong law of large numbers for iterates of weakly contractive in mean random-valued functions
 17:10–17:30 Kazuki Okamura: On expectations of power means of random variables
- 17:35–17:55 Problems and Remarks

Dinner 18:00-20:00

February 4, Tuesday

1^{st} Morning Session – Chair: Zsolt Páles

09:00-09:20	Michał Lewandowski: Biseparable representations of certainty equiva-
	lents – an introduction
00 05 10 05	

09:25–10:05 Jacek Chudziak: Biseparable representations of certainty equivalents

Coffee Break 10:10–10:40

2ND MORNING SESSION – CHAIR: JACEK CHUDZIAK

10:40-11:00	Miklós Laczkovich: <i>Operations on</i> \mathbb{R}^n <i>isomorphic to</i> (\mathbb{R}^n , +), <i>and the</i>
	resultant of forces
11:05-11:25	Harald Fripertinger: On n-associative formal power series over rings
11:30-11:50	Wojciech Jabłoński: On substitution in rings of formal power series

Lunch 12:00-13:00

Afternoon Session – Chair: Ioan Raşa

15:00-15:20	Janusz Matkowski: Weak associativity: new examples and open ques-
	tions
15:25-15:45	Dorota Głazowska: Invariance problems for generalized classical
	weighted means
15:50-16:10	Problems and Remarks

Coffee Break 16:10–16:40

Festive Dinner at University of Debrecen 17:00-

February 5, Wednesday

1 ^{sт}	Morning	Session -	CHAIR:	Lajos	Molnár
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09:00-09:40	Che Tat Ng: Measuring movement of incomes
09:45-10:05	Maciej Sablik: Social indices and bisymmetry

Coffee Break 10:10–10:40

 2^{ND} Morning Session – Chair: Maciej Sablik

10:40-11:00	Włodzimierz Fechner: Two characterizations of quasiconvexity
11:05-11:25	Ana-Maria Acu: Types of convexity related to a C_0 -semigroup

11:30–11:50 Paweł Pasteczka: *Equality and comparison of generalized quasiarithmetic means*

Lunch 12:00–13:00

1st Afternoon Session – Chair: Dorian Popa

15:00-15:20	Borbála Fazekas: On the characterization of turbulent magnetohydro-
	dynamic mean flows
15:25-15:45	Chaitanya Gopalakrishna: Iterative roots of multifunctions
15:50-16:10	Diana Otrocol: Fixed point results for non-self operators on \mathbb{R}^m_+ -metric
	spaces

Coffee Break 16:15-16:45

2nd Afternoon Session – Chair: Gian Luigi Forti

- 16:45–17:05 Lajos Molnár: Maps on positive cones in operator algebras preserving relative entropies
 17:10–17:30 Jacek Chmieliński: Norm derivatives in complex spaces
- 17:35–17:55 Problems and Remarks

Dinner 18:00-20:00

February 6, Thursday

1st Morning Session – Chair: Miklós Laczkovich

09:00-09:40	Zsolt Páles: A short CV of János Aczél
09:45-10:05	László Losonczi: Products and inverses of multidiagonal matrices with
	equally spaced diagonals

Coffee Break 10:10–10:40

 2^{nd} Morning Session – Chair: Che Tat Ng

10:40-11:00	Justyna Sikorska: Is it still a characterization of the logarithmic mean?
11:05-11:25	László Horváth: Majorization type inequalities using signed measures
11:30-11:50	Tomasz Szostok: Orderings on measures induced by higher-order
	monotone functions

Lunch 12:00-13:00

1st Afternoon Session – Chair: Wolfgang Förg-Rob

15:00-15:20	Zoltán Boros: From space perception to utility: the evolution of some
	functional equations
15:25-15:45	Péter Tóth: Regularity preservation for quasisums
15:50-16:10	Miklós Pintér: Functional equations in cooperative game theory

Coffee Break 16:15-16:45

2ND Afternoon Session – Chair: Zoltán Boros

16:45-17:05	José María Almira: Solution to a conjecture of Laird and McCann
17:10-17:30	Dorian Popa: On the best Ulam constant of a linear differential opera-
	tor
17:35-17:55	Problems and Remarks dedicated to personal memories about János
	Aczél
17:55-18:00	Closing

Dinner 18:00-20:00

List of Participants

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List of Talks

- 1. ANA-MARIA ACU, Types of convexity related to a C₀-semigroup
- 2. JOSÉ MARÍA ALMIRA, Solution to a conjecture of Laird and McCann
- 3. KAROL BARON, Strong law of large numbers for iterates of weakly contractive in mean randomvalued functions
- 4. ZOLTÁN BOROS, From space perception to utility: the evolution of some functional equations
- 5. JACEK CHMIELIŃSKI, Norm derivatives in complex spaces
- 6. JACEK CHUDZIAK, Biseparable representations of certainty equivalents
- 7. JEAN DHOMBRES, How to write a book with János Aczél?
- 8. BORBÁLA FAZEKAS, On the characterization of turbulent magnetohydrodynamic mean flows
- 9. WŁODZIMIERZ FECHNER, Two characterizations of quasiconvexity
- 10. GIAN LUIGI FORTI, C^2 solutions of an alternative quadratic functional equation
- 11. HARALD FRIPERTINGER, On n-associative formal power series over rings
- 12. Roman GER, Erdős number
- 13. ATTILA GILÁNYI, On a functional equation arising from investigations of utility
- 14. DOROTA GŁAZOWSKA, Invariance problems for generalized classical weighted means
- 15. CHAITANYA GOPALAKRISHNA, Iterative roots of multifunctions
- 16. László HORVÁTH, Majorization type inequalities using signed measures
- 17. WOJCIECH JABŁOŃSKI, On substitution in rings of formal power series
- 18. GERGELY KISS, Two theorems of Aczél and continuity
- 19. MIKLÓS LACZKOVICH, Operations on \mathbb{R}^n isomorphic to $(\mathbb{R}^n, +)$, and the resultant of forces
- 20. MICHAŁ LEWANDOWSKI, Biseparable representations of certainty equivalents an introduction
- 21. László LOSONCZI, Products and inverses of multidiagonal matrices with equally spaced diagonals
- 22. JANUSZ MATKOWSKI, Weak associativity: new examples and open questions
- 23. LAJOS MOLNÁR, Maps on positive cones in operator algebras preserving relative entropies
- 24. CHE TAT NG, Measuring movement of incomes
- 25. KAZUKI OKAMURA, On expectations of power means of random variables
- 26. DIANA OTROCOL, Fixed point results for non-self operators on \mathbb{R}^{m}_{+} -metric spaces
- 27. ZSOLT PÁLES, Contributions of János Aczél to the theory of means
- 28. PAWEŁ PASTECZKA, Equality and comparison of generalized quasiarithmetic means
- 29. MIKLÓS PINTÉR, Functional equations in cooperative game theory
- 30. DORIAN POPA, On the best Ulam constant of a linear differential operator
- 31. IOAN RAŞA, Functional equations related to Appell polynomials and Heun functions (II)
- 32. MACIEJ SABLIK, Social indices and bisymmetry
- 33. JUSTYNA SIKORSKA, Is it still a characterization of the logarithmic mean?
- 34. László SZÉKELYHIDI, Convolution-Type Functional Equations
- 35. TOMASZ SZOSTOK, Orderings on measures induced by higher-order monotone functions
- 36. Péter TÓTH, Regularity preservation for quasisums

Abstracts

Ana-Maria Acu

("Lucian Blaga" University of Sibiu)

Types of convexity related to a C_0 -semigroup

(joint work with Georgian Chivu, Ioan Raşa)

Let *K* be a convex compact subset of \mathbb{R}^p , $p \ge 1$, having nonempty interior. Starting with a suitable positive linear projection *T* defined on C(K), Altomare and Raşa defined in [1] the weakly *T*-convex functions. Using *T*, a C_0 -semigroup of operators on C(K) was constructed and the generalized *A*-subharmonic functions were defined, where *A* is the infinitesimal generator of the semigroup. It was proved that if a function is weakly *T*-convex, then it is generalized *A*-subharmonic. The authors of [1] conjectured that the converse is also true, but as far as we know this is still an open problem. We present some results related to the conjecture. Namely, starting with the conjecture, we prove that a suitable stronger hypothesis entails a stronger conclusion. This study extends certain results from [1].

- A.M. Acu, I. Raşa, *Generalized Subharmonic and Weakly Convex Functions*, In: A.M. Candela, M. Cappelletti Montano, E. Mangino (eds.) Recent Advances in Mathematical Analysis, Trends in Mathematics, Birkhäuser, Cham, 2023.
- [2] F. Altomare, I. Raşa, Feller semigroups, Bernstein type operators and generalized convexity associated with positive projections, New Developments in Approximation Theory (Dortmund, 1998), Internat. Ser. Numer. Math. 132, Birkhäuser, Basel, 1999, 9–32.

José María Almira

(University of Murcia)

Solution to a conjecture of Laird and McCann

Let X_d denote indistinctly either the space $\mathcal{D}(\mathbb{R}^d)'$ of Schwartz complex valued distributions defined on \mathbb{R}^d or the space $C(\mathbb{R}^d)$ of continuous complex valued functions defined on \mathbb{R}^d . For $f \in \mathcal{D}(\mathbb{R}^d)'$ we introduce the operators

 $\tau_h(f)\{\phi\} = f\{\tau_{-h}(\phi)\}$, where $h \in \mathbb{R}^d$ and $\phi \in \mathcal{D}(\mathbb{R}^d)$ is any test function,

and $\tau_{-h}(\phi)(x) = \phi(x - h)$. We also consider the operators

$$O_P(f)\{\phi\} = \frac{1}{|\det(P)|} f\{O_{P^{-1}}(\phi)\},\$$

where $P \in \mathbf{GL}_d(\mathbb{E})$ is any invertible matrix, $\phi \in \mathcal{D}(\mathbb{R}^d)$ is any test function, and

$$O_{P^{-1}}(\phi)(x) = \phi(P^{-1}x) \quad \text{ for all } x \in \mathbb{R}^d$$

In the talk we will demonstrate the following result, which generalizes a theorem by Loewner [3, 4] to distributions and whose proof solves a conjecture posed by Laird and McCann [2] in 1984.

Theorem Let $d \ge 2$ be a natural number, let $f \in X_d$ and assume that, for a certain finite dimensional space $V \subseteq X_d$ we have that:

- $f \in V$
- *V* is translation invariant (i.e., $\tau_h(V) \subseteq V$ for all $h \in \mathbb{R}^d$).
- *V* is invariant under orthogonal transformations of \mathbb{R}^d (i.e., $O_P(V) \subseteq V$, for all $P \in \mathbf{O}(d)$).

Then f is, in distributional sense, an ordinary polynomial on \mathbb{R}^d . In particular, f is equal almost everywhere to an ordinary polynomial and, if f is a continuous ordinary function, then it is an ordinary polynomial.

- [1] J. M. Almira, On Loewner's characterization of polynomials, Jaen J. Approx. 8 (2) (2016), 175–181.
- [2] P.G. Laird, R. McCann, On some characterizations of polynomials, Amer. Math. Monthly 91 (2) (1984), 114-116.
- [3] P.G. Laird, On characterizations of exponential polynomials, Pacific J. Math. 80 (1979), 503-507.
- [4] C. Loewner, On some transformation semigroups invariant under Euclidean and non-Euclidean isometries, J. Math. Mech. 8 (1959), 393-409.

Karol Baron

(University of Silesia in Katowice)

Strong law of large numbers for iterates of weakly contractive in mean random-valued functions

(joint work with Rafał Kapica)

Assume $(\Omega, \mathcal{A}, \mathbb{P})$ is a probability space, (X, ρ) is a complete and separable metric space with the σ -algebra \mathcal{B} of all its Borel subsets and $f: X \times \Omega \to X$ is measurable for $\mathcal{B} \otimes \mathcal{A}$ and such that

$$\int_{\Omega} \rho(f(x,\omega), f(z,\omega)) \mathbb{P}(d\omega) \le \beta(\rho(x,z)) \quad \text{for } x, z \in X$$

with a concave $\beta : [0, \infty) \to [0, \infty)$ satisfying $\sum_{n=1}^{\infty} \beta^n(t) < \infty$ for $t \in (0, \infty)$, and

$$\int_{\Omega} \rho(f(x_0, \omega), x_0) \mathbb{P}(d\omega) < \infty \quad \text{for an } x_0 \in X.$$

We consider the sequence of iterates of f defined on $X \times \Omega^{\mathbb{N}}$ by $f^0(x, \omega) = x$ and

$$f^n(x,\omega) = f(f^{n-1}(x,\omega),\omega_n) \text{ for } n \in \mathbb{N},$$

its weak limit π^f and the problem of the almost sure convergence

$$\lim_{n \to \infty} \frac{1}{n} \sum_{k=1}^{n} \psi \circ f^{k}(x, \cdot) = \int_{X} \psi d\pi^{f}$$

for $x \in X$ and Lipschitzian $\psi : X \to \mathbb{R}$.

Zoltán Boros

(University of Debrecen)

From space perception to utility: the evolution of some functional equations

In an axiomatic development of the psychophysical theory of binocular vision [3], some invariance relations led to functional equations of the form

(1)
$$F(x + \theta t, y + \sigma t) = H_{\theta,\sigma}(F(x, y), t)$$

where the real valued function *F* is defined on a given bounded, convex, and open subset of the plane, $\theta, \sigma \in \{-1, 0, 1\}$, and the functions $H_{\theta,\sigma}$ are also unknown. Assuming that *F* can be represented as the sum of two continuous and strictly monotone real functions and it fulfills a pair of the considered functional equations (1), we proved that *F* is linear.

Motivated by these results, the present author [4] investigated pairs of equations

$$[(1), (\theta, \sigma) \in \{(1, 0), (0, 1)\}]$$

for a continuous function $F : S \to \mathbb{R}$, strictly monotone in one of its variables, defined on an arbitrary connected and open set $S \subseteq \mathbb{R}^2$. In this case, every solution has the representation

(2)
$$F(x,y) = \varphi(ax + by) \qquad ((x,y) \in S),$$

where *a* and *b* are non-zero real numbers and φ is a strictly monotone function defined on an interval. Péter Tóth [6] has extended the latter result to continuous functions in several variables (relaxing the monotonicity assumption as well). Applying the representation theorem for continuous, associative and cancellative operations on an interval ([1], [2] and [5]), he could replace the addition on the lefthand side with particular operations, obtaining characterizations of various particular utility functions in terms of systems of functional equations [7].

- [1] J. Aczél, Sur les opérations définies pour nombres réels, Bull. Soc. Math. France 76 (1949), 59-64.
- [2] J. Aczél, *Lectures on Functional Equations and Their Applications*, Dover Publications, Inc., Mineola, New York, 1966.
- [3] J. Aczél, Z. Boros, J. Heller, and C. T. Ng, *Functional Equations in Binocular Space Perception*, J. Math. Psych. 43 (1) (1999), 71–101.
- [4] Z. Boros, *Systems of generalized translation equations on a restricted domain*, Aequationes Math. 67 (2004), 106–116.
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Jacek Chmieliński

(University of the National Education Commission, Krakow)

Norm derivatives in complex spaces

In a *real* normed space X the norm derivatives at $x \in X$ in the direction of $y \in X$ are defined as

$$\rho'_{\pm}(x,y) := \lim_{\lambda \to 0^{\pm}} \frac{\|x + \lambda y\|^2 - \|x\|^2}{2\lambda} = \|x\| \lim_{\lambda \to 0^{\pm}} \frac{\|x + \lambda y\| - \|x\|}{\lambda}.$$

Now, if *X* is a *complex* normed space, then for each $\theta \in [0, 2\pi)$ we define

$$\rho_{\theta}'(x,y) = \lim_{r \to 0^+} \frac{\|x + re^{i\theta}y\|^2 - \|x\|^2}{2r} = \|x\| \lim_{r \to 0^+} \frac{\|x + re^{i\theta}y\| - \|x\|}{r}.$$

In particular, for $\theta \in \{0, \pi\}$, we have $\rho'_0 = \rho'_+$ and $\rho'_\pi = -\rho'_-$. Thus the notion of complex norm derivatives ρ'_{θ} extends that of ρ'_{\pm} . We will speak on the properties and applications of the functionals ρ'_{θ} .

Jacek Chudziak

(University of Rzeszów)

Biseparable representations of certainty equivalents

(joint work with Michał Lewandowski)

Assume that (x, y; p) is a binary monetary lottery that pays x with probability p, and y with probability 1 - p. Let F(x, y; p) denote the certainty equivalent of (x, y; p), i.e. a certain amount, the receipt of which is as good for the decision maker as playing the lottery. We investigate individual preferences that lead to the following representation of the certainty equivalent

(1)
$$F(x,y;p) = u^{-1}(w(p)u(x) + (1 - w(p))u(y)),$$

where $u : \mathbb{R} \to \mathbb{R}$ is a continuous strictly increasing utility function and $w : [0, 1] \to [0, 1]$ is a probability weighting function.

We consider various axioms leading to (1). In particular, we characterize (1) in the domain of all binary lotteries as well as in the subset of simple lotteries, where one of the two payouts of a lottery is fixed. Furthermore, in the domain of all binary lotteries we separately characterize the case where F(x, y; p) depends on the rank of the payouts (rank dependency) and the case where it does not. Finally we present an analysis of the relationships between the proposed axioms and several versions of the bisymmetry-like axiom.

Jean Dhombres

(Centre Alexandre-Koyré)

How to write a book with János Aczél?

My aim, in responding with pleasure to the invitation to speak on the occasion of the hundred-first anniversary of the birth of János Aczél, is to evoke an extraordinary character, particularly for those who were unable to know him, and who continue to nourish this field that he so well illustrated, functional equations, and the journal that he founded, Aequationes mathematicae. I had the opportunity to meet him often, in Canada for sure, and in many other places, and of course to spend time with him on the occasion of a book we wrote together, which came out in 1989 as number 31 of the Encyclopedia of mathematics and its applications: Functional equations in several variables. The book was dedicated to our seven grand-children. Just to provide you with some context, it was the year that I also wrote a rather thick history book in French with my wife: Birth of a New Power: Science and Scientists in France, 1793-1824. I am not embarrassed to revive these events, and perhaps to let you feel the spirit of a bygone world, but I know in advance that I will miss the verve, the witticisms, all the dazzling vitality of János who chained together jokes, sometimes very profound. I will do my best.

Borbála Fazekas

(University of Debrecen)

On the characterization of turbulent magnetohydrodynamic mean flows

(joint work with József Kolumbán)

We consider the ideal magnetohydrodynamic system

 $\partial_t u + \operatorname{div}(uu^T - BB^T) + \nabla p = 0$ $\partial_t B + \operatorname{div}(Bu^T - uB^T) = 0$ $\operatorname{div} u = \operatorname{div} B = 0,$

where $u, B: \mathbb{R}^3 \times (0, T) \to \mathbb{R}^3$, $p: \mathbb{R}^3 \times (0, T) \to \mathbb{R}$.

We are aiming at infinitely many weak solutions of this system via the so-called Tartar framework. For this purpose the Λ -convex hull of the set

$$K_{r,s} = \{(u, S, B, E) \in \mathbb{R}^{15} \mid S = uu^T - BB^T + pI, E = B \times u, \\ ||u + B|| = r, ||u - B|| = s, p \in \mathbb{R}, |p| \le rs\}$$

with $r, s \in \mathbb{R}$ fixed, is of main importance, where the cone Λ consists of elements $(\xi_x, \xi_t) \in \mathbb{R}^4 \setminus \{0\}$ with $\xi_t u + \xi_x \cdot S = 0$, $\xi_t B + \xi_x \times E = 0$, $\xi_x \cdot u = 0$, $\xi_x \cdot B = 0$.

We determine the first laminate of $K_{r,s}$ and give an enclosing set for the Λ -convex hull of $K_{r,s}$.

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Włodzimierz Fechner

(Lodz University of Technology)

Two characterizations of quasiconvexity

Let X be a real linear space, $D \subset X$ a nonempty convex set, and $\overline{\mathbb{R}} = \mathbb{R} \cup \{-\infty, +\infty\}$. A function $f: D \to \overline{\mathbb{R}}$ is called quasiconvex if

$$\forall_{x,y \in D} \forall_{t \in [0,1]} f(tx + (1-t)y) \le \max\{f(x), f(y)\}.$$

This talk focuses on characterizing quasiconvexity under assumptions of radial lower semicontinuity or upper semicontinuity. Drawing inspiration from the work of Páles [3], Daróczy and Páles [1] on convexity characterizations (see also Leonetti [2]), we establish, among others, the following result: *Assume that* $f: D \rightarrow \mathbb{R}$ *is a radially lower semicontinuous function. If*

 $\forall_{x,y \in D} \exists_{z \in]x,y[} f(z) \le \max\{f(x), f(y)\},\$

then f is quasiconvex on D.

We will show some applications of our findings, including an extension of Sion's minimax theorem [4] and characterizations of quasiconvex risk measures.

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Gian Luigi Forti

(University of Milan)

C^2 solutions of an alternative quadratic functional equation

The aim of this presentation is the investigation of the following alternative quadratic functional equation:

$$f(x+y) + f(x-y) - 2f(x) - 2f(y) \neq 0 \quad \text{implies} \\ g(x+y) + g(x-y) - 2g(x) - 2g(y) = 0,$$

where $f, g : \mathbb{R} \to \mathbb{R}$ are $C^2(\mathbb{R})$ functions. The assumed regularity condition allows a useful integral representation of the quadratic difference of the two functions f and g, indeed we have

(1)
$$f(x) = \int_0^x \tau(t)dt + \frac{C_1}{2}x^2 + C_2x + C_3, \quad C_3 = f(0),$$
$$g(x) = \int_0^x \sigma(t)dt + \frac{D_1}{2}x^2 + D_2x + D_3, \quad D_3 = g(0),$$

where τ and σ are C^1 functions on \mathbb{R} , with $\tau(0) = \tau'(0) = 0 = \sigma(0) = \sigma'(0)$.

Under these conditions we obtain the following result:

Theorem. The alternative quadratic equation has only trivial solutions, that is either $f(x) = \alpha x^2$ for $x \in \mathbb{R}$, or $g(x) = \beta x^2$ for $x \in \mathbb{R}$, for some real numbers α and β .

Some open problems are then presented.

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Harald Fripertinger

(University of Graz)

On *n*-associative formal power series over rings

(joint work with Susan F. El-Deken)

In [1] *n*-associative formal power series over \mathbb{C} were studied. This was a generalization of associativity, which is the special case of 2-associativity and was investigated in [2]. These results on 2-associativity were generalized by F. Halter–Koch in [3] for formal power series in two variables over commutative rings. The starting point for all these investigations had been Hazewinkel's book [4] in which formal group laws are studied over \mathbb{Q} -algebras. Now we present some results on *n*-associative formal power series over a commutative ring *R* with 1 following Halter-Koch's ideas.

A formal power series $F(x_1, ..., x_n) \in R[[x_1, ..., x_n]]$ in *n* variables, $n \ge 3$, of order at least 1 is called *n*-associative, if the following $\binom{n}{2}$ equations

$$F(F(x_1,\ldots,x_n),x_{n+1},\ldots,x_{2n-1}) = \ldots = F(x_1,\ldots,x_{n-1},F(x_n,x_{n+1},\ldots,x_{2n-1}))$$

hold true. The sequence $\varphi_i(x) = F(0^{i-1}, x, 0^{n-i}), i \in \{1, ..., n\}$, where *x* stands in the *i*th position, plays an important role in the study of *n*-associative series.

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Roman Ger

(University of Silesia)

Erdős number

Due to Professor **János Aczél** and the paper [1], my Erdős number is 2. Like numerous members of this meeting I am very proud of that fact. I do believe that it justifies the idea of evoking some details of [1] showing that the problem of determining all utility measures over binary gambles that are both separable and additive leads to the functional equation

$$f(v) = f(vw) + f[vQ(w)], \quad v, vQ(w) \in [0, k), w \in [0, 1].$$

The following conditions seem to be more or less natural to that problem: f is strictly increasing, Q is strictly decreasing, both map their domains onto intervals (f onto a [0, K), Q onto [0, 1]), thus both are continuous, k > 1, f(0) = 0, f(1) = 1, Q(1) = 0, Q(0) = 1. We determine, however, the general solution without any of these conditions (except $f : [0, k) \longrightarrow \mathbb{R}_+ := [0, \infty)$, $Q : [0, 1] \longrightarrow \mathbb{R}_+$, both into). If we exclude two trivial solutions, then we get as the general solution $f(v) = \alpha v^{\beta}$ ($\beta > 0$, $\alpha > 0$; $\alpha = 1$ for f(1) = 1), which satisfies all the above conditions.

The paper concludes with a remark on the case where the equation is satisfied only almost everywhere.

In the second part of my talk I take the liberty of telling the audience some of my less formal contacts with János (like in [2]).

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Attila Gilányi

(University of Debrecen)

On a functional equation arising from investigations of utility

This talk focuses on the study of the functional equation

(1) $F_1(t) - F_1(t+s) = F_2[F_3(t) + F_4(s)],$

which appears in several investigations related to utility (e.g., in [4], [5] and [7]). We also consider some further interesting equations arising during the solution of (1) (cf., [1], [2] and [6]), as well as related problems, particularly the "Program" of studying extensions established by János Aczél ([3]).

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Dorota Głazowska

(University of Zielona Góra, Poland)

Invariance problems for generalized classical weighted means

Let $I \subset \mathbb{R}$ be an interval and let $f: I \to \mathbb{R}$ and $id|_I - f$ be strictly increasing functions. Then the bivariable function $A_f: I^2 \to \mathbb{R}$ given by

$$A_f(x, y) = f(x) + y - f(y)$$

is a strict mean in the interval *I*. The mean A_f generalizes the classical weighted arithmetic means. Now let $I \subset (0, +\infty)$ be an interval. Let $g: I \to (0, +\infty)$ and $h: I \to (0, +\infty)$ be strictly increasing functions such that the functions $\frac{\operatorname{id}_I}{g}$ and $\operatorname{id}_I - h$ are also increasing. Then the functions $G_g: I^2 \to \mathbb{R}$ and $H_h: I^2 \to \mathbb{R}$ given, respectively, by

$$G_g(x,y) = \frac{g(x)}{g(y)}y, \quad H_h(x,y) = \frac{xy}{x - h(x) + h(y)},$$

are strict means in the interval *I*. These means are natural generalizations, respectively, of the classical weighted geometric mean and the classical weighted harmonic mean.

During the talk, the solutions of some invariance problems involving these means will be presented. Among others, we consider invariance of a generalized weighted arithmetic mean A_f with respect to a generalized weighted arithmetic mean-type mapping (A_a, A_h) , i.e. we solve the equation

$$A_f \circ \left(A_g, A_h \right) = A_f,$$

under some additional regularity assumptions on the unknown functions f, g, h. The form of the generalized weighted geometric mean allows to apply these results, to obtain the relevant solutions of the following invariance equation

$$G_f \circ \left(G_g, G_h \right) = G_f$$

involving only generalized weighted geometric means.

Finally, under general conditions ensuring that A_f , G_g and H_h are means, assuming that f and h are twice continuously differentiable, g is three times continuously differentiable in $I \subset (0, \infty)$, we examine when the equality

$$G_g \circ \left(H_h, A_f \right) = G_g,$$

holds, i.e. when the mean G_g is invariant with respect to the mean-type mapping (H_h, A_f) . This is a report on the research made jointly with Janusz Matkowski.

Chaitanya Gopalakrishna

(Indian Statistical Institute)

Iterative roots of multifunctions

In this talk we discuss some new results on the nonexistence of iterative roots of multifunctions on arbitrary sets. First, we introduce a fundamental class of multifunctions, called pullback multifunctions, and show that the iterative root problem for such multifunctions can be reduced to the corresponding problem for single-valued maps. Then, focusing on multifunctions that are not pullbacks, we prove that typically if the graph of a multifunction has a special point with a relatively large number of paths leading to it then such a multifunction does not admit any iterative root. Notably, we impose no constraints on the number of set-valued points of the map under consideration. We also illustrate our results with some examples, particularly showing the nonexistence of iterative roots of some specified orders for certain complex polynomials and their pullbacks. The talk is based on our recent work [1].

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László Horváth

(University of Pannonia)

Majorization type inequalities using signed measures

In this talk we derive majorization type integral inequalities using measure spaces with signed measures. We obtain necessary and sufficient conditions for the studied integral inequalities to be satisfied. To apply our results, we first generalize Hardy-Littlewood-Pólya and Fuchs inequalities. Then we deal with the nonnegativity of some integrals with nonnegative convex functions. As a consequence, the known characterization of Steffensen-Popoviciu measures on compact intervals is extended to arbitrary intervals. Finally, we give necessary and sufficient conditions for the satisfaction of the integral Jensen inequality and the integral Lah-Ribarič inequality for signed measures. All the considered problems are also studied for special classes of convex functions.

Wojciech Jabłoński

(Jan Kochanowski University of Kielce)

On substitution in rings of formal power series

It is known that the set $R[[x_1, ..., x_n]]$ of all formal power series of *n* variables $x_1, ..., x_n$ over a commutative ring R is a ring with the usual operations of addition and multiplication of formal power series. Moreover, with no additional assumptions one can define the operation of substitution in the ideal $m = (x_1, ..., x_n)$. Following results from [1-3] we discuss the existence of a substitution in the ring of formal power series of several variables over a ring with a metric. We show that the sufficient and necessary condition for the existence of a substitution which appears in the mentioned papers is natural and almost obvious also in the ring of formal power series in several variables.

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Gergely Kiss

(Corvinus University of Budapest)

Two theorems of Aczél and continuity

(joint work with Patricia Szokol and Pál Burai)

In this talk, I will first present two renowned theorems of Aczél concerning the characterization of quasi-arithmetic means and quasi-sums. Then, I will introduce and discuss our simplified characterization of quasi-arithmetic means without assuming continuity, and I will formulate several potentially open questions posed in our first joint paper [4]. Additionally, I will discuss some dichotomy results arising from the relaxation of symmetry and reflexivity, which provide more nuanced descriptions of quasi-arithmetic means. I will also present a construction that, among other significant consequences, demonstrates that continuity in Aczél's theorem on quasi-sums might not be eliminated. Finally, I will outline some open research directions.

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Miklós Laczkovich

(Eötvös Loránd University)

Operations on \mathbb{R}^n isomorphic to $(\mathbb{R}^n, +)$, and the resultant of forces

Let \oplus be a binary operation on \mathbb{R}^n . When will (\mathbb{R}^n, \oplus) be isomorphic to the Abelian group $(\mathbb{R}^n, +)$? An obvious necessary condition is that (\mathbb{R}^n, \oplus) is a commutative semigroup with unit.

We show that, assuming the condition above and $n \ge 3$, the following is sufficient: \oplus commutes with the elements of SO_n (the family of orthogonal transformations of \mathbb{R}^n with determinant 1); that is, $(Aa) \oplus (Ab) = A(a \oplus b)$ for every $a, b \in \mathbb{R}^n$ and $A \in SO_n$.

The result is closely connected to the characterization of vector addition given by D'Alembert, Darboux and others.

Michał Lewandowski

(Warsaw School of Ecoomics)

Biseparable representations of certainty equivalents - an introduction

(joint work with Jacek Chudziak)

We consider the following biseparable representation of the certainty equivalent:

(1)
$$F(x,y;p) = u^{-1}(w(p)u(x) + (1 - w(p))u(y)),$$

where (x, y; p) is a binary monetary prospect, u is a utility function, and w is a probability weighting function. We characterize this model in the domain of all binary prospects, as well as in a subset of simple prospects, i.e. prospects in which one of the two payoffs is fixed. For both domains, we consider the case of a general probability weighting function w, corresponding to the certainty equivalent in the rank-dependent utility model, as well as several others, e.g.

- a. a rank-independent model, in which w is self-conjugate,
- b. an anticipated utility model, in which w satisfies the condition w(0.5) = 0.5,
- c. an expected utility model, in which w is the identity function.

This talk will serve as an introduction and motivation for the plenary talk by Jacek Chudziak, who will focus on our theoretical results concerning the characterization of these different models. I will show why the models we characterize are so important and outline how our results and approach can help with issues related to model identification and distinguishing it from the process of model validation in the context of experimental methods for preference elicitation. I will show that the preferences generating certainty equivalents of binary prospects in the form (1) are very general. In particular, they are consistent with the phenomenon of preference reversal. Even assuming that preferences for prospects are represented by certainty equivalents of prospects, they encompass many popular nonexpected preference models. By focusing on model (1), we investigate the "common denominator" of all these models. I will also outline the problem of cross-domain model extensions.

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László Losonczi

(University of Debrecen)

Products and inverses of multidiagonal matrices with equally spaced diagonals

Let *n*, *k* be fixed natural numbers with $1 \le k \le n$ and let $A_{n+1,k,2k,\dots,sk}$ denote a complex $(n + 1) \times (n + 1)$ multidiagonal matrix having s = [n/k] sub- and superdiagonals at distances $k, 2k, \dots, sk$ from the main diagonal. We prove that the set $\mathcal{MD}_{n,k}$ of all such multidiagonal matrices is closed under multiplication and powers with positive exponents. Moreover the subset of $\mathcal{MD}_{n,k}$ consisting of all nonsingular matrices is closed under taking inverses and powers with negative exponents. In particular, we obtain that the inverse of a nonsingular matrix $A_{n+1,k}$ (called *k*-tridiagonal) is in $\mathcal{MD}_{n,k}$, moreover if $n+1 \le 2k$ then $A_{n+1,k}^{-1}$ is also *k*-tridiagonal. Using this fact, we give an explicit formula for this inverse.

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Janusz Matkowski

(University of Zielona Góra)

Weak associativity: new examples and open questions

In a recent paper [1] it is observed that every weighted quasiarithmetic mean is *weakly associative*, i.e. it satisfies the equality

 $M\left(M\left(x,y\right),x\right)=M\left(x,M\left(y,x\right)\right),$

and is not a symmetric function. In the present talk we present a new class of weakly associative means and propose some open questions.

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Lajos Molnár

(University of Szeged)

Maps on positive cones in operator algebras preserving relative entropies

We consider several concepts of (not only numerical valued) relative entropies on positive cones in C^* algebras. We present recent (not yet published) results showing that surjective transformations between positive cones that preserve any of those quantities necessarily originate from Jordan *-isomorphisms between the underlying full algebras. In the special case of matrix algebras, we consider the problem of relaxing the condition of surjectivity.

Che Tat Ng

(University of Waterloo)

Measuring movement of incomes

Professor Aczél has shown us the usefulness of functional equations in the theory of measurements. The axioms involved in the characterization and derivation of a measure may be qualitative or quantitative in nature. Functional inequalities also play their role. I shall bring up some of them which are found in the literature for discussion.

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Kazuki Okamura

(Shizuoka University)

On expectations of power means of random variables

(joint work with Yoshiki Otobe)

In general, it is hard to compute or characterize the expectation of a power mean of independent and identically distributed random variables, due to the fractional power of the mean. We investigate expectations of power means of complex-valued random variables by using fractional calculus. We deal with both negative and positive orders of the fractional derivatives. We explicitly compute the expectations of the power means for both the univariate Cauchy distribution and the Poincaré distribution on the upper half-plane. We show that for these distributions the expectations are invariant with respect to the sample size and the value of the power. This talk will depend on [1].

References

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Diana Otrocol

(Technical University of Cluj-Napoca)

Fixed point results for non-self operators on \mathbb{R}^m_+ -metric spaces

(joint work with Adela Novac and Veronica Ilea)

The purpose of this paper is to study some problems of the fixed point theory for non-self operators on \mathbb{R}^m_+ -metric spaces. The results complement and extend some known results given in the papers [1]-[3].

- [1] A. Chis-Novac, R. Precup, I.A. Rus, *Data dependence of fixed points for non-self generalized contractions*, Fixed Point Theory, 10 (1) (2009), 73–87.
- [2] V. Ilea, D. Otrocol, I.A. Rus, M.A. Serban, *Applications of fibre contraction principle to some classes of functional integral equations*, Fixed Point Theory, 23 (1) (2022), 279–292.
- [3] I.A. Rus, Some variants of contraction principle, generalizations and applications, Stud. Univ. Babeş-Bolyai Math., 61 (3) (2016), 343–358.

Zsolt Páles

(University of Debrecen)

Contributions of János Aczél to the theory of means

One of the most significant results of the theory of quasiarithmetic means, their characterization theorem, was discovered independently by de Finetti [4], Kolmogorov [5] and Nagumo [6] in 1930-31. Each of these characterizations involved quasiarithmetic means with non-fixed number of variables. János Aczél's main result was published [1] in 1947, where he introduced the notion of bisymmetry which turned out to be the central concept for the characterization of quasiarithmetic means with fixed number of variables. This paper was the starting point of a new research direction in the theory of means and had many applications in aggregation and decision theory.

One of the important generalizations of quasiarithmetic means was introduced by Bajraktarević [3] in 1963. Motivated by the properties of the so-called Rényi entropies and their applications in information theory, the characterization of homogeneity of these means became a fundamental question which was answered by Aczél and Daróczy [2] in 1963.

In the talk, we aim to describe the above mentioned results in details and sketch some of their applications.

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- [2] J. Aczél and Z. Daróczy, Über verallgemeinerte quasilineare Mittelwerte, die mit Gewichtsfunktionen gebildet sind, Publ. Math. Debrecen 10 (1963), 171–190.
- [3] M. Bajraktarević, Sur une généralisation des moyennes quasilinéaires, Publ. Inst. Math. (Beograd) (N.S.) 3 (17) (1963), 69–76.
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- [5] A.N. Kolmogorov, Sur la notion de la moyenne, Rend. Accad. dei Lincei 12 (6) (1930), 388–391.
- [6] M. Nagumo, Über eine Klasse der Mittelwerte, Japanese J. Math. 7 (1930), 71–79.

Paweł Pasteczka

(University of the National Education Commission, Krakow)

Equality and comparison of generalized quasiarithmetic means

(joint work with Zsolt Páles)

The purpose of this talk is to extend the definition of quasiarithmetic means by taking a strictly monotone generating function instead of a strictly monotone and continuous one. We establish the properties of such means and compare them to the analogous properties of standard quasiarithmetic means. The comparability and equality problems of generalized quasiarithmetic means are also solved. We also provide an example of a mean which, depending on the underlying interval or on the number of variables, could be or could not be represented as a generalized quasiarithmetic mean.

References

[1] Zs. Páles, P. Pasteczka, *Equality and comparison of generalized quasiarithmetic means*, preprint, *arXiv:2412.07315* (math.CA).

Miklós Pintér

(Corvinus University of Budapest)

Functional equations in cooperative game theory

The talk aims to provide a concise introduction to the (potential) applications of functional equations and inequalities in the theory of transferable utility (TU) cooperative games [6]. The class of TU-games is a collection of finite-dimensional vector spaces, and the so-called solutions defined on this class (or a subclass) of TU-games. A solution is a set-valued mapping that assigns a "solution"—a subset of a finite-dimensional vector space—to each game. In particular, a value is a single-valued solution. The most well-known solutions include the Shapley value [4], the core [1,5], and the nucleolus [3], among others.

To argue for or against a particular solution, the solution is characterized using so-called axioms. A characterization of a solution has the form: A FUNCTION defined on a specific SUBCLASS of TUgames is the SOLUTION if and only if it satisfies a given set of AXIOMS. In such cases, we say the AXIOMS characterize the SOLUTION on the SUBCLASS. These axioms take the form of functional equations and inequalities. In other words, a characterization result asserts that a set of functional equations and inequalities has a particular unique solution over a given domain.

For example, the Shapley value, which is a linear function, has numerous axiomatizations on various subclasses of TU-games. While (almost) each axiomatization is both interesting and significant, only three are considered "fundamentally" distinct: the one employing the linearity axiom [4], the one without the linearity axiom [7], and a recursive one [2].

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Dorian Popa

(Technical University of Cluj-Napoca)

On the best Ulam constant of a linear differential operator

(joint work with Alina-Ramona Baias)

The linear differential operator with constant coefficients

$$D(y) = y^{(n)} + a_1 y^{(n-1)} + \ldots + a_n y, \quad y \in C^n(\mathbb{R}, X)$$

acting in a Banach space X is Ulam stable if and only if its characteristic equation has no roots on the imaginary axis. We prove that if the characteristic equation of D has distinct roots r_k satisfying Re $r_k > 0$, $1 \le k \le n$, then the best Ulam constant of D is

$$K_D = \frac{1}{|V|} \int_0^\infty \Big| \sum_{k=1}^n (-1)^k V_k e^{-r_k x} \Big| dx,$$

where $V = V(r_1, r_2, ..., r_n)$ and $V_k = V(r_1, ..., r_{k-1}, r_{k+1}, ..., r_n)$, $1 \le k \le n$, are Vandermonde determinants.

Ioan Raşa

(Technical University of Cluj-Napoca)

Functional equations related to Appell polynomials and Heun functions (II)

(joint work with Ana-Maria Acu)

We are concerned with the functional equation

(1)
$$(x^2 - x)H'_n(x) = n(2x - 1)(H_n(x) - H_{n-1}(x)), \ n \ge 1,$$

where $x \in \mathbb{R}$ and $H_0(x) = 1$.

It was investigated (see [1], [2], and the references therein) in relation with Legendre polynomials, Appell polynomials, Heun functions and certain entropies.

In this paper we consider functions of the form

$$H_n(x) = \sum_{k=0}^n b_{n,k} (x^2 - x)^k, \ n \ge 0,$$

where $x \in \mathbb{R}$, $b_{n,k} \in \mathbb{R}$, $b_{n,0} = 1$, $n \ge 0$.

We want to determine conditions on the coefficients $b_{n,k}$ under which $H_n(x)$ is

- a) a solution to (1),
- b) a Heun function,
- c) a hypergeometric function.

Moreover, we study the functions $H_n(x)$ in connection with Appell polynomials, Gegenbauer (ultraspherical) polynomials and certain entropies.

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Maciej Sablik

(University of Silesia)

Social indices and bisymmetry

The Human Poverty Index is one of measures used by United Nations to compare countries of the world. It is admitted that roughly speaking the HPI is a power mean. However, there has been no consistency in choosing the power α . We show that the consistency may be obtained by solving the generalized bisymmetry equation. We also do recall a result on representing the bisymmetrical operations in some function spaces.

Justyna Sikorska

(University of Silesia)

Is it still a characterization of the logarithmic mean?

Starting from the famous Hermite-Hadamard inequality for φ -convex functions and a characterization of the logarithmic mean we present some regularity results and outcomes on comparison of means. Continuing the considerations from [1], we weaken the conditions and see whether the characterization remains valid.

References

[1] T. Nadhomi, M. Sablik, J. Sikorska, On a characterization of the logarithmic mean, Results Math. 79 (2024), 200.

László Székelyhidi

(University of Debrecen)

Convolution-Type Functional Equations

In this talk we present some fundamental results in spectral synthesis – in terms of functional equations. We show that the basic problems – solved and unsolved – in spectral synthesis are closely related to the theory of convolution-type functional equations.

Tomasz Szostok

(University of Silesia)

Orderings on measures induced by higher-order monotone functions

(joint work with Zsolt Páles)

Let $I \subset \mathbb{R}$ be an interval. For a function $f: I \to \mathbb{R}$, for $n \in \mathbb{N} \cup \{0\}$ and for all pairwise distinct elements $x_0, \ldots, x_n \in I$, we define

$$f[x_0,...,x_n] := \sum_{j=0}^n \frac{f(x_j)}{\prod_{k=0,k\neq j}^n (x_j - x_k)},$$

which is called the *nth-order divided difference* for f at x_0, \ldots, x_n . We say that f is *n-increasing* if, for all pairwise distinct elements $x_0, \ldots, x_n \in I$, we have that $f[x_0, \ldots, x_{n+1}] \ge 0$ (if the opposite inequality is true then we say that f is *n-decreasing*). Let μ be a non-zero bounded signed Borel measure on [0, 1]. We give a sufficient condition under which the inequality

(1)
$$\int_{[0,1]} f((1-t)x + ty) d\mu(t) \ge 0, \qquad x, y \in I \text{ with } x < y,$$

is fulfilled for all functions f which are simultaneously k_1 -increasing (or decreasing), k_2 -increasing (or decreasing), ..., k_l -increasing (or decreasing) for given nonnegative integers k_1, \ldots, k_l . The necessary condition is also investigated. These results are applied to study the continuous solutions of linear functional inequalities of the form

$$\sum_{i=1}^k a_i f(\alpha_i x + (1 - \alpha_i) y) \ge 0$$

where $\sum_{i=1}^{k} a_i = 0$ and of the inequality

$$\frac{1}{y-x}\int_x^y f(t)dt \leq \sum_{i=1}^k a_i f(\alpha_i x + (1-\alpha_i)y),$$

with $\sum_{i=1}^{k} a_i = 1$.

Péter Tóth

(University of Debrecen)

Regularity preservation for quasisums

Let $n \ge 2$ be an integer and, for k = 1, ..., n, let $f_k : I_k \longrightarrow \mathbb{R}$ be a continuous, strictly monotone function defined on a nonempty open interval. Moreover, suppose that the function $g : f_1(I_1) + \cdots + f_n(I_n) \longrightarrow \mathbb{R}$ is also continuous, strictly monotone. Then the mapping $F : I_1 \times \cdots \times I_n \longrightarrow \mathbb{R}$ defined by

$$F(x_1,...,x_n) = g(f_1(x_1) + \dots + f_n(x_n)) \qquad (x_1 \in I_1,...,x_n \in I_n).$$

is called a quasisum.

In our talk we will show that if the quasisum F is continuously differentiable (in Fréchet sense) then each of the generator functions g, f_1, \ldots, f_n are continuously differentiable as well. We will also present that the same holds for higher order continuous differentiability: if $p \in \mathbb{N}$ and F is p-times continuously differentiable, then g, f_1, \ldots, f_n are p-times continuously differentiable real functions.